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Letter

Electromagnetic form factors of the nucleon in chiral perturbation theory including vector mesons

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Abstract. We calculate the electromagnetic form factors of the nucleon up to fourth order in manifestly Lorentz-invariant chiral perturbation theory with vector mesons as explicit degrees of freedom. A systematic power counting for the renormalized diagrams is implemented using both the extended on-mass-shell renormalization scheme and the reformulated version of infrared regularization. We analyze the electric and magnetic Sachs form factors, G_E and G_M, and compare our results with the existing data. The inclusion of vector mesons results in a considerably improved description of the form factors. We observe that the most dominant contributions come from tree-level diagrams, while loop corrections with internal vector meson lines are small.

PACS. 12.39.Fe Chiral Lagrangians – 13.40.Gp Electromagnetic form factors

1 Introduction

Experiments on elastic electron-nucleon scattering provide the most fundamental information on the electromagnetic structure of the nucleon [1]. In the one-photon-exchange approximation, this information is contained in four electromagnetic form factors, two each for the proton and the neutron, which parameterize the single-nucleon matrix element of the electromagnetic current operator $J^\mu(x)$:

$$\langle N(p')|J^\mu(0)|N(p)\rangle = \bar{u}(p')\left[\gamma^\mu F_1^N + \frac{i\sigma^{\mu\nu}q_\nu}{2m_p}F_2^N\right]u(p),$$

where $q = p' - p$ and $N = p, n$. The so-called Dirac and Pauli form factors $F_1^N(Q^2)$ and $F_2^N(Q^2)$ are functions of $Q^2 = -q^2 \geq 0$ and are normalized such that, at $Q^2 = 0$, they reduce to the electric charge and the anomalous magnetic moment in units of the elementary charge and the nuclear magneton $e/(2m_p)$, respectively:

$$F_1^p(0) = 1,\ F_1^n(0) = 0,\ F_2^p(0) = 1.793,\ F_2^n(0) = -1.913.$$

For the analysis of experimental data it is more convenient to use the electric and magnetic Sachs form factors $G_E^N(Q^2)$ and $G_M^N(Q^2)$ [2] which are related to the Dirac and Pauli form factors via

$$G_E^N(Q^2) = F_1^N(Q^2) - \frac{Q^2}{4m_N^2}F_2^N(Q^2),$$
$$G_M^N(Q^2) = F_1^N(Q^2) + F_2^N(Q^2).$$

Their Fourier transforms in the Breit frame can be related to the distribution of charge and magnetization inside the nucleon. These form factors have been the aim of extensive research and, for the case of the proton, are known over a wide momentum range. An apparent inconsistency of the results for the ratio of the electric and magnetic proton form factors as obtained from the Rosenbluth separation in comparison with those from the polarization transfer method has recently been addressed in terms of two-photon exchange corrections [3]. Due to the lack of a free neutron target, the neutron form factors are not as well known. However, recent experiments using polarized beams and/or targets have improved our knowledge, especially of G_E^n (for an overview and a recent discussion of the existing form factor data, see refs. [4–6] and references therein). Given the wealth and precision of available data, the description of the electromagnetic form factors presents a stringent test for any theory or model of the strong interaction.

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Chiral perturbation theory (ChPT) [7–9] is the effective field theory of quantum chromodynamics in the low-energy region (for a recent review, see ref. [10]). Using ChPT, the form factors have been calculated within the early relativistic approach [9], heavy-baryon ChPT [11, 12], and the small-scale expansion [13]. The spectral functions of the isovector electromagnetic form factors of the nucleon have been analyzed at the one- and two-loop order in refs. [14] and [15], respectively. More recently, also two manifestly Lorentz-invariant renormalization schemes, namely infrared regularization (IR) of ref. [16] and the extended on-mass-shell (EOMS) scheme of ref. [17], have been used to calculate the form factors up to and including order $\mathcal{O}(q^4)$ [18,19]. The results in the two renormalization schemes are very similar, but fail to describe both proton form factors G_E^p and G_M^p as well as the magnetic neutron form factor G_M^n for momentum transfers beyond $Q^2 \sim 0.1\,\mathrm{GeV}^2$. To improve these results higher-order contributions have to be included. This can be achieved by performing a full calculation at $\mathcal{O}(q^5)$ which would also include the analysis of two-loop diagrams. That such a calculation in a manifestly Lorentz-invariant framework is, at least in principle, possible has been demonstrated in ref. [20].

Another possibility is to include additional degrees of freedom, through which some of the higher-order contributions are re-summed. This latter approach is less systematic and proceeds more along the lines of phenomenological models. It has long been established that vector mesons play an important role in the description of the nucleon form factors, and by including them dynamically in the effective field theory one hopes to generate the most important higher-order contributions. In ref. [18] the ρ, ω, and ϕ mesons have been included in the calculation. One finds that the vector mesons re-sum important higher-order contributions and the obtained description of form factors up to $Q^2 \approx 0.4\,\mathrm{GeV}^2$ is satisfactory. Since diagrams with internal vector meson lines *inside* loops cannot be treated within the original formulation of infrared regularization, such diagrams have not been considered in ref. [18].

The EOMS renormalization scheme of ref. [17] and the reformulated version of infrared regularization of ref. [21] both allow to include virtual vector mesons *systematically* in the region of the applicability of baryon chiral perturbation theory [22]. The standard power counting determines which diagrams (including diagrams with vector mesons appearing in loops) should be taken into account to a given order in the chiral expansion. In this Letter we present the nucleon electromagnetic form factors to order $\mathcal{O}\left(q^4\right)$ in the framework of baryon ChPT with explicit vector mesons. Our calculations contain *all* diagrams which appear to this order in the chiral expansion.

2 Effective Lagrangian and power counting

The Lagrangian needed for calculating the electromagnetic form factors up to order q^4 without explicit vector mesons can be found in ref. [19]. Here, q collectively stands for a small quantity such as the pion mass, small external four-momenta of the pion and small external three-momenta of the nucleon. In this Letter, we consider, in addition, the ρ, ω, and ϕ mesons as explicit degrees of freedom. We make use of the vector-field representation of ref. [23], in which the ρ meson is represented by $\rho_\mu = \rho_\mu^a \tau^a$ and the ω and ϕ mesons by ω_μ and ϕ_μ, respectively. The coupling of the vector mesons to pions and external fields is at least of the order q^3,

$$\mathcal{L}_{\pi V}^{(3)} = -f_\rho \mathrm{Tr}(\rho^{\mu\nu} f_{\mu\nu}^{+}) - f_\omega \omega^{\mu\nu} f_{\mu\nu}^{(s)} - f_\phi \phi^{\mu\nu} f_{\mu\nu}^{(s)} + \ldots, \quad (1)$$

where

$$f_{\mu\nu}^{(s)} = \partial_\mu v_\nu^{(s)} - \partial_\nu v_\mu^{(s)},$$

$$f_{\mu\nu}^{+} = u f_{\mu\nu}^{L} u^{+} + u^{+} f_{\mu\nu}^{R} u,$$

with

$$f_{\mu\nu}^{L} = \partial_\mu l_\nu - \partial_\nu l_\mu - i\left[l_\mu, l_\nu\right],$$

$$f_{\mu\nu}^{R} = \partial_\mu r_\nu - \partial_\nu r_\mu - i\left[r_\mu, r_\nu\right].$$

The $SU(2)$ matrix $u^2 = U$ contains the pion fields. For the case of a coupling to an external electromagnetic potential $\mathcal{A}_\mu$ the external fields are given by $r_\mu = l_\mu = -e\tau_3 \mathcal{A}_\mu/2$ and $v_\mu^{(s)} = -e\mathcal{A}_\mu/2$ ($e^2/4\pi \approx 1/137$, $e > 0$) [10]. Furthermore

$$V_{\mu\nu} = \nabla_\mu V_\nu - \nabla_\nu V_\mu, \quad V = \rho, \omega, \phi$$

with

$$\nabla_\mu V_\nu = \partial_\mu V_\nu + [\Gamma_\mu, V_\nu]$$

and

$$\Gamma_\mu = \frac{1}{2}\left[u^\dagger, \partial_\mu u\right] - \frac{i}{2} u^\dagger r_\mu u - \frac{i}{2} u l_\mu u^\dagger.$$

Only those terms that are used for the calculation of the form factors up to order q^4 are given here; a complete list of possible interaction terms at order q^3 can be found in ref. [23].

The lowest-order Lagrangian for the coupling to the nucleon is given by

$$\mathcal{L}_{NV}^{(0)} = \frac{1}{2} \sum_{V=\rho,\omega,\phi} g_V \bar{\Psi} \gamma^\mu V_\mu \Psi, \quad (2)$$

and the $\mathcal{O}(q)$ Lagrangian reads

$$\mathcal{L}_{NV}^{(1)} = \frac{1}{4} \sum_{V=\rho,\omega,\phi} G_V \bar{\Psi} \sigma^{\mu\nu} V_{\mu\nu} \Psi. \quad (3)$$

Finally, each renormalized diagram has a chiral order D which is determined with the following power counting rules in addition to the ones in ref. [19]: vertices from $\mathcal{L}_{\pi V}^{(3)}$ count as $\mathcal{O}(q^3)$ and vertices from $\mathcal{L}_{NV}^{(i)}$ as $\mathcal{O}(q^i)$, respectively, while the vector-meson propagators count as $\mathcal{O}(q^0)$.

Table 1. Values of the vector-meson coupling constants within an $\mathcal{O}(q^4)$ calculation.

f_ρ	f_ω	f_ϕ	g_ρ	g_ω	g_ϕ	G_ρ (GeV^{-1})	G_ω (GeV^{-1})	G_ϕ (GeV^{-1})
0.10	0.03	0.05	4.0	42.8	−20.6	13.0	0.96	−3.3

Table 2. Values of the relevant low-energy constants in the EOMS and the infrared regularization scheme. The LECs c_i are given in units of GeV^{-1}, the d_i in units of GeV^{-2}, and the e_i in units of GeV^{-3}.

	c_2	c_4	$\tilde{c_6}$	$\tilde{c_7}$	d_6	d_7	e_{54}	e_{74}
EOMS	2.66	2.45	1.26	−0.13	1.21	1.30	−0.76	1.65
IR	2.66	2.45	0.47	−1.87	0.98	0.24	−0.26	−0.90

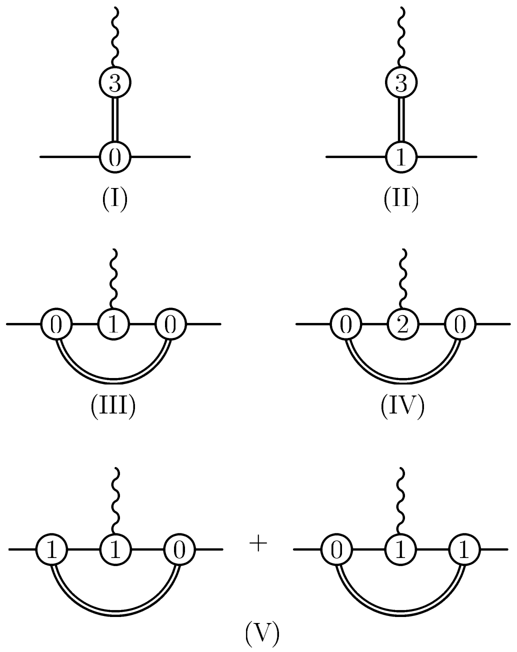

Fig. 1. Feynman diagrams including vector mesons that contribute to the electromagnetic form factors of the nucleon up to and including $\mathcal{O}(q^4)$. External leg corrections are not shown. Solid, wiggly, and double lines refer to nucleons, photons, and vector mesons, respectively. The numbers in the interaction blobs denote the order of the Lagrangian from which they are obtained. The direct coupling of the photon to the nucleon is obtained from $\mathcal{L}_{\pi N}^{(1)}$ and $\mathcal{L}_{\pi N}^{(2)}$.

3 Results and discussion

The relevant diagrams that do *not* contain vector mesons and their explicit contributions to the form factors are given in ref. [19]. The additional diagrams involving vector mesons that contribute in the calculation of the form factors up to and including order $\mathcal{O}(q^4)$ using the Lagrangians of eqs. (1)-(3) are shown in fig. 1. To renormalize the results we employed both the infrared regularization [16] in its reformulated version [21] as well as the EOMS scheme of ref. [17]. Since the results for the vector-meson diagrams contain only loop integrals with no internal pion lines, these loop contributions vanish in the infrared regularization and only the tree graphs (I) and (II) of fig. 1 contribute. On the other hand, in the EOMS scheme the renormalized diagram (III) contributes at $\mathcal{O}(q^3)$ and the renormalized diagrams (IV) and (V) at $\mathcal{O}(q^4)$, respectively. Finally, at the given order the vector-meson diagrams do not contribute to the wave function renormalization constant Z.

In order to obtain the form factors numerically we have to fix the parameters of the Lagrangian. The parameters of the vector-meson Lagrangian of eq. (1) for the coupling to external fields have been taken from ref. [23], and those of eqs. (2) and (3) for the coupling of vector mesons to the nucleon from the dispersion relations of refs. [24,25]. The numerical values of these coupling constants are given in table 1.

To determine the low-energy constants (LECs) c_2, c_4, $\tilde{c_6}$, $\tilde{c_7}$, d_6, d_7, e_{54}, and e_{74} of the πN effective Lagrangian [26] we proceed analogously to ref. [19]. We use the more recent values of ref. [25] for the proton electric and magnetic radii, $r_E^p = 0.848\,\text{fm}$ and $r_M^p = 0.857\,\text{fm}$, and the neutron magnetic radius, $r_M^n = 0.879\,\text{fm}$. Table 2 summarizes the values of the LECs in the EOMS and infrared regularization schemes. The differences between the LECs in the respective renormalization schemes originate in the different treatment of loop integrals. In comparison to a calculation without vector mesons only the parameters d_i and e_i change.

The results for the Sachs form factors in the momentum transfer region $0\,\text{GeV}^2 \leq Q^2 \leq 0.4\,\text{GeV}^2$ are shown in fig. 2. For comparison, fig. 3 contains the corresponding results at $\mathcal{O}(q^4)$ without vector mesons. As expected on phenomenological grounds, the quantitative description of the data has improved considerably for $Q^2 \gtrsim 0.1$ GeV^2. The small difference between the two renormalization schemes is due to the way the regular higher-order terms of loop integrals are treated. Note that on an absolute scale the differences between the two schemes are comparable for both G_E^p and G_E^n. Numerically, the results are similar to those of ref. [18]. Due to the renormalization condition, the contribution of the vector-meson loop diagrams either vanishes (IR) or turns out to be small (EOMS). Thus, in hindsight our approach puts the traditional phenomenological vector-meson dominance model on a more solid theoretical basis. We would like to emphasize that, in the

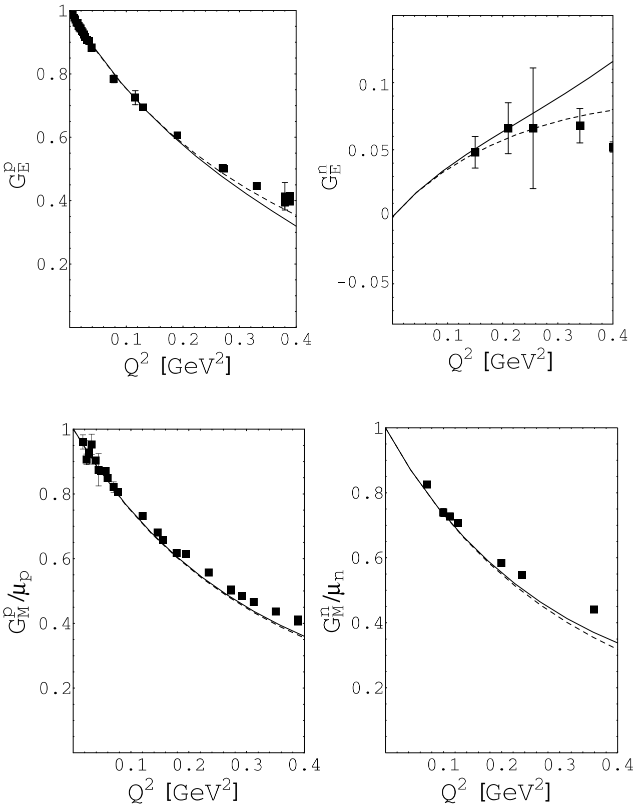

Fig. 2. The Sachs form factors of the nucleon in manifestly Lorentz-invariant chiral perturbation theory at $\mathcal{O}(q^4)$ including vector mesons as explicit degrees of freedom. Full lines: results in the extended on-mass-shell scheme; dashed lines: results in infrared regularization. The experimental data are taken from ref. [6].

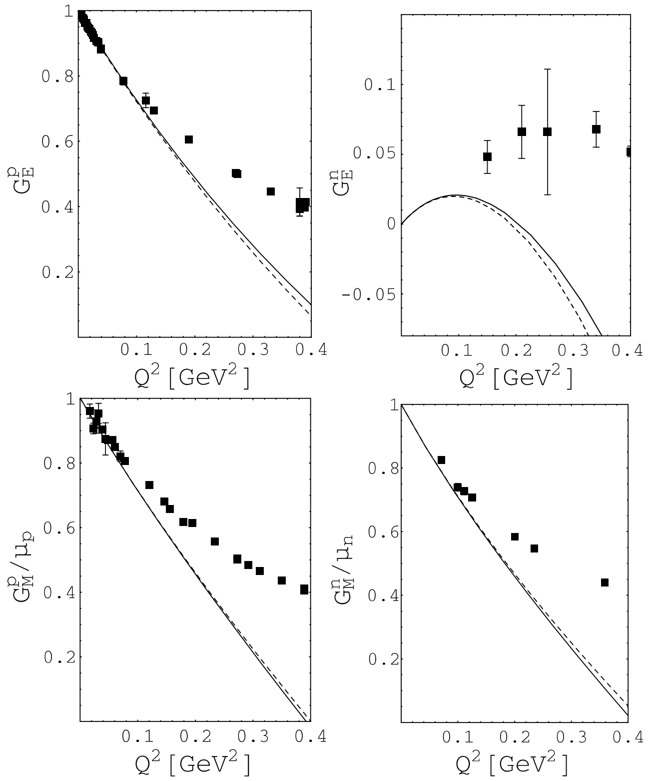

Fig. 3. The Sachs form factors of the nucleon in manifestly Lorentz-invariant chiral perturbation theory at $\mathcal{O}(q^4)$ without vector mesons. Full lines: results in the extended on-mass-shell scheme; dashed lines: results in infrared regularization. The experimental data are taken from ref. [6].

sense of a strict chiral expansion in terms of small external momenta q and quark masses m_q at a fixed ratio m_q/q^2 [8], up to and including $\mathcal{O}(q^4)$, the results with and without explicit vector mesons are completely equivalent. The additional vector-meson contributions up to this order are compensated by a readjustment of the low-energy constants pertaining to the theory including vector mesons as dynamical degrees of freedom. On the other hand, the inclusion of vector-meson degrees of freedom in the present framework results in a reordering of terms which, in an ordinary chiral expansion, would show up at higher orders q^5 and q^6 etc. It is these terms which change the form factor results favorably for larger values of Q^2. It should be noted, however, that this re-organization proceeds according to well-defined rules so that a controlled, order-by-order, calculation of corrections is made possible. In contrast to the calculation without vector mesons, the Sachs form factors G_E^p, G_M^p, and G_M^n now show sufficient curvature to generate a more accurate phenomenology for values of Q^2, where the ordinary chiral expansion to the same order is no longer reliable. Also the description of G_E^n has improved considerably when compared to a calculation without the inclusion of vector mesons.

To conclude, we have shown how the traditional, but *ad hoc* and phenomenological vector-meson dominance model can be incorporated consistently into an effective field theory approach. Using a suitable renormalization condition we were able to set up a systematic power counting and to justify that the vector-meson loop contributions are suppressed while the tree-level pole diagrams generate the well-known important contributions to the nucleon electromagnetic form factors.

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